On the time-to-depth conversion velocity for radar scanning of shallow freshwater lakes (0.3 – 2 m).

Introduction

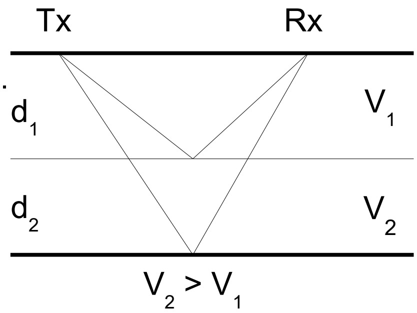
Shallow lakes in urban surroundings often serve aesthetic purposes as well as being buffer zones for rain water where biologic and non-biologic sediments can settle before the water runs to the recipient. In this respect, years of sedimentation may degrade the value of the lake and the lake owners are often facing huge costs when restoration becomes imminent in order to have the lakes serving their intended purposes. Regardless of the restoration methods chosen, a good starting point would be to obtain a detailed bathymetric map of the lake in order to calculate volumetry so as to gain insight into the condition of the lake. That can be done by radar scanning the lake, but as with much geophysical field work, what seems to be routine can quickly become anything but routine when faced with practical problems.
As an example, a Secchi-disk was used to measure the visibility in the water and also used for measuring the water depth in the lake which was infested with algae. In addition, particles in the water made it impossible to see the bottom of the lake for depths greater than 35 cm. Dirt and leaves had accumulated at the bottom forming a transition zone from water to uncompressed sediment. Since this zone has only a small reflection coefficient, the bottom of the lake was difficult to identify on the radar images and for that reason the measurements of true depth were taken to support the interpretations. In order to match the true depth with the interpretations of the lake bottom, the conversion velocity had to be given what seemed to be an unrealistically high value in the water. Apart from finding a good conversion velocity it must also be noted that there is uncertainty regarding the 'true depth' measurements, because the transition zone at the bottom made it difficult to determine the very soft lake bottom with the Secchi-disk when poor visibility prevented the operator from seeing the contact point in depth.
In principle, the conversion of two-way reflection time of radar signals obtained from radar scanning, along with high quality GPS data, to depths is z = ½vt, where z is the depth, t is the interpreted reflection time and v is a velocity. On land this is done almost routinely and often without considerations as to the particulars regarding inhomogeneities along the recording paths. However, on a lake the signals travel in homogeneous media in the vicinity of the antenna, and one should therefore think no special considerations should be given to the conversion velocity and that the theoretical velocity of radio waves in freshwater can be used directly. Unfortunately, practical experience shows that this is not always the case for scanning on shallow freshwater lakes.

Prerequisites

The following considerations deal with a pure ray-analogy in piece-wise homogeneous horizontal layers. The interval velocity, v_{int}, is the constant propagation speed of the radar signal within each layer and with a straight ray path.

T is the two-way travel time to a layer boundary, and for a single layer we get

(1) $$T^2_{x,1} = T^2_{0,1} + \frac{x^2}{v^2}$$ where $v = v_{int}$ and x is the Tx-Rx distance.



If there are more than one layer and we assume straight ray paths as shown on the right, we can still use this formula, but v will now be a function of the layer velocities. If v_i denotes the interval velocity in the i'th layer, d_i is the thickness of the layer i and t_i is the two-way travel time in the i'th layer, then the vertical two-way travel time to layer n can be expressed as

(2) $$T_{0,n} = 2\sum_{i=1}^{n} \frac{d_i}{v_i}$$

For an n-layer subsurface we get

(3) $$T^2_{x,n} = T^2_{0,n} + \frac{x^2}{v^2_{a,n}}$$ where v_a is the **average** velocity down to layer n.

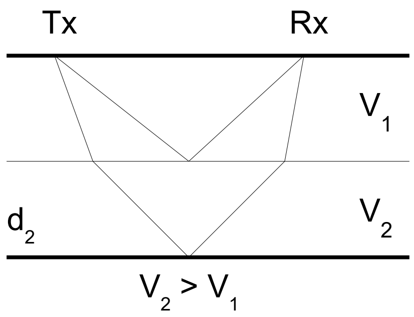
where v_{ave} is given as

(4) $$v_{a,n} = \frac{\sum_{i=1}^{n} v_i t_i}{T_{0,n}} = \frac{2\sum_{i=1}^{n} d_i}{T_{0,n}}$$

The above equations are based on the assumption of the shortest ray path in space. If we use the principle of the shortest ray path in time, things get more complicated.
It can be shown ([1], Taner and Koehler, 1969) that this leads to the following expression:

(5) $$T^2_{x,n} = T^2_{0,n} + \frac{x^2}{v^2_{RMS,n}}$$ + higher order terms in x

(6) $$v^2_{RMS,n} = \frac{\sum_{i=1}^{n} v_i^2 t_i}{T_{0,n}}$$



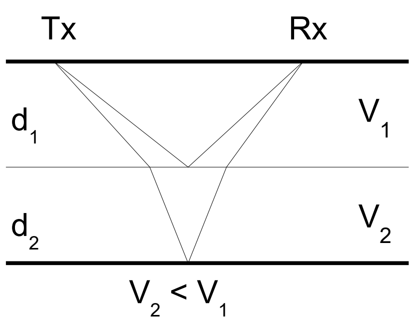
In [1] it is noted that if the higher order terms after the two as given above are neglected, the error is of the order of only 2% for horizontal layers, which for the vast majority of applications, is acceptable. The squared RMS velocity is a time-weighted average of the squared interval velocities.

Some practical considerations

In order to get around on a shallow lake and to sail close to obstacles in the lake, lake borders and vegetation, a small inflatable boat is often used to carry equipment and the operator. Many of the inflatables have a small keel in order to stabilize the movements of the boat. Furthermore, if heavy equipment, such as lead-batteries must be onboard to power the trolling motor, it is preferable that there is a flat deck made of plywood at the bottom of the boat, so the antenna box can be placed on the deck. There is a problem in determining exactly where the antenna elements are located with respect to the water surface. The position changes when the operator enters the boat. The 'true depth' is usually measured from the water surface. It can also be difficult to determine how much air-filled space there is between the antenna bottom and the exact position of the antenna elements within the antenna box is usually unknown.
These considerations lead to the introduction of an unknown delay of the radar signals, and the geometry can be viewed as an artificial layer of unknown characteristics with respect to thickness and interval velocity. These problems may vary depending on the type of boat and the antenna as well as the operator's weight.

If the overall characteristics of the layer between the contact surface of the water with the bottom of the boat and the antenna is considered to be a single layer, then we have a two-layer model from the antenna to the bottom of the lake. The upper layer is constant in terms of thickness and interval velocity and the second layer is the varying depth to the bottom of the lake.

On the right, the figure shows the ray path in the two layers with velocities at either extreme of the velocities found in many textbook tables; layer one consists mostly of air ($3.0 \ 10^8$ m/s) and in layer two freshwater ($0.333 \ 10^8$ m/s).



Problem discussion

Equations (4) and (6) can be plotted as a function of the depth from the bottom of the boat to the bottom of the lake, as shown in fig. 1. The layer model is given in table 1 below.

Thickness of the upper layer:	10 cm
Velocity in the upper layer:	$2.8\ 10^8$ m/s
Velocity in the lower layer:	$0.33\ 10^8$ m/s

Table 1.

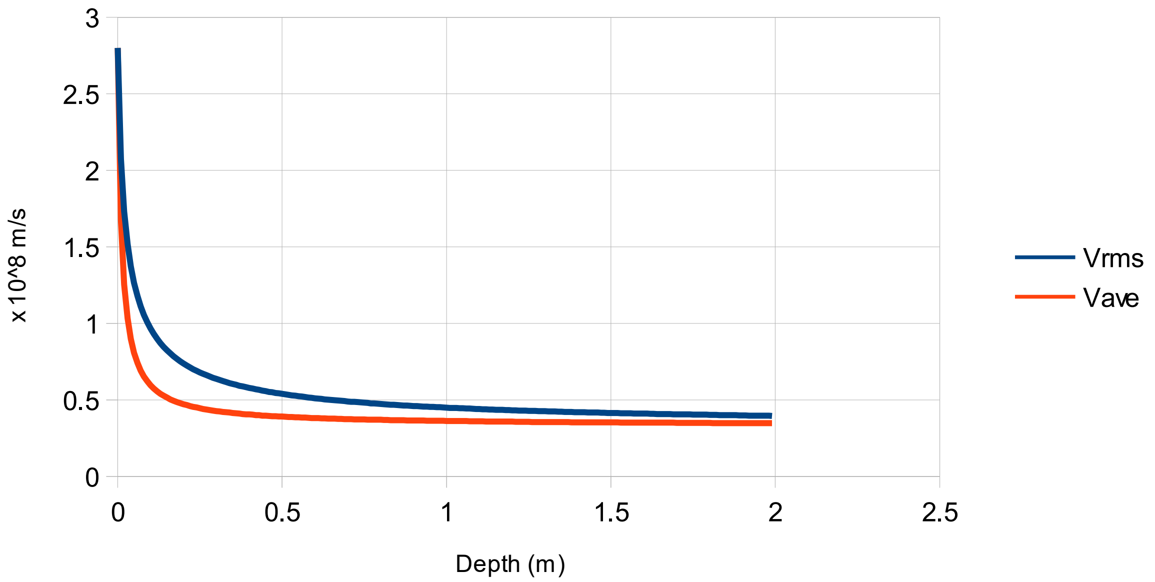


Figure 1. Thickness of the upper layer is 10 cm.

Fig. 1 shows that as we go down from the bottom of the boat (the interface between the two layers), the velocities drop quickly because of the very high velocity contrast.

The upper layer has a thickness of 10 cm. If this thickness is reduced to 7 cm while keeping the velocity contrast as given in table 2, we get the velocity-depth curves in fig. 2.

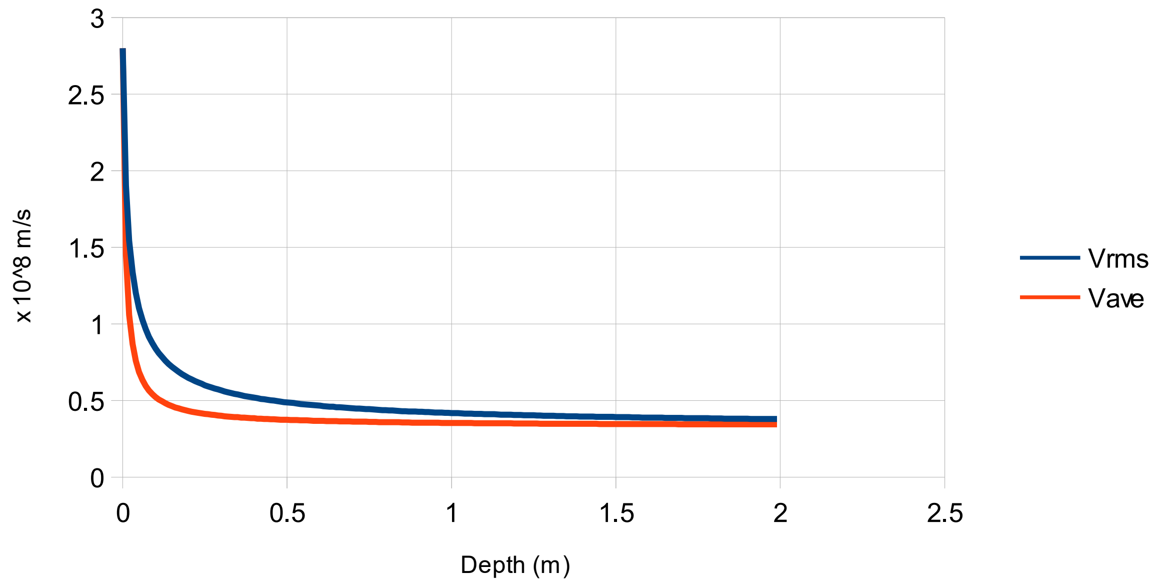


Figure 2. Thickness of the upper layer is 7 cm.

By reducing the upper layer thickness by 3 cm, this difference will impact the time-to-depth conversion velocity as shown in table 2.

	Upper layer: 10 cm		Upper layer: 7 cm	
Depth (m)	V_{rms} x10^8 m/s	V_{ave} x10^8 m/s	V_{rms} x10^8 m/s	V_{ave} x10^8 m/s
0.30	0.738	0.472	0.648	0.432
0.50	0.578	0.405	0.519	0.384
1.00	0.460	0.366	0.427	0.356
1.50	0.420	0.354	0.396	0.348

Table 2.

½ $\Delta V_{ave} T_0$	Depth
2.49 cm	0.30
2.57 cm	0.50
2.61 cm	1.00
2.62 cm	1.50

Table 3. The difference in depth estimates with 7 and 10 cm of a mostly air-filled upper layer.

Table 2 shows the velocity values at selected depths. If we use V_{ave} as the time-to-depth conversion velocity directly at selected depths and assume the upper layer thickness is 10 cm while it was only 7 cm, we get table 3. The error is less than the difference in thickness, but it goes to show that even a few cm of uncertainty regarding the thickness of the bottom of the inflatable boat is significant.

The antenna used is a 250 MHz shielded antenna with an antenna element separation of 35 cm. With a wavelength of 1.2 m in air, it is impossible to infer the upper layer dimensions from the radar data thus preventing us from finding interval velocities using Dix' formula.

From fig. 1 and table 2 it seems to be the case that the time-to-depth conversion velocity depends on the water depth and on the thickness of the upper layer.

Equation (5) is used for finding the NMO-correction velocities ([2], Booth A. D. et al., 2011, and the references therein) and as such it is not the time-to-depth conversion velocity.

The following procedure may resolve the problem to a certain degree:

Firstly, apply an NMO-correction based on the antenna element separation and an RMS-velocity model which resembles figs.1 and 2.
Secondly, use an average velocity model which for depths greater than 30 cm is not altogether wrong in order to get a linear depth scale on the radar images.

The rationale behind this approach is that the NMO-correction brings the lake bottom closer to the water surface for shallow depths (< 1 m) and it is now two-way vertical reflection time. Subsequently, by using a constant time-to-depth conversion velocity matching the flat parts of the V_{ave} curves in figs. 1 and 2, a reasonable and linear depth scale can be used. With thicknesses of 10 and 7 cm for the upper layer, time-to-depth conversion velocities of 0.362 10^8 m/s and 0.354 10^8 m/s, respectively, can be used. These velocities are calculated at 1 m depth.

Alternatively, the time-to-depth velocity must be given as a function of depth, which will imply a non-linear depth scale or radargram stretching in time.

However, the problem of determining the position of the antenna elements and getting the correct true water depth measurements still remains. These issues will be the subject of future field-procedure development.

References

[1] Taner M. T. and Koehler F., 1969, Velocity spectra – digital computer derivation and applications of velocity function. Geophysics, Vol. 34, No. 6.

[2] Booth A. D., Clark R. A. and Murray T., Influences on the resolution of GPR velocity analyses and a Monte Carlo simulation for establishing velocity precision, Near Surface Geophysics, 2011, **9**, 399-411.